

Quantum simulation of microscopic gravitational waves with a two-level system

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On the background of the Born-Oppenheimer (adiabatic) approximation, we investigate the geometrical and topological structure in the theory of quantum gravitation by use of the path integral method and half-classical approximation. As we know, Berry curvature can be extracted from the linear response of a driven two-level system to nonadiabatic manipulations of its Hamiltonian. In parameter space of the Hamiltonian, magnetic monopoles can be artificially simulated. Ripples occur in Hilbert space when the monopole travels from inside to outside the surface of energy manifold spanned by system parameters. From this point, we set up the connection between the ripples characterized by the fidelity of quantum states in Hilbert space and gravitational waves in microscopic scales. This may open a window for the study of geometrical and topological properties of quantum gravity with the help of qubit systems.

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1. INTRODUCTION

It is recognized that quantum theory and general relativity are two pillars of modern physics. However, the goal of finding a consistent theory of quantum gravity still remains elusive [1]. In spite of this, there still exists many ways to stimulate and guide researchers to solve problems of quantum cosmology, and one may find out the vulnerable spot of the difficulties in quantum cosmology through some already known characters of quantum gravity [2–10].

Quantum gravity in path integral form was significantly stimulated by the superspace approach to the canonical quantization of gravity [3, 4]. A profound formulation of quantum mechanics of quantization of a system is based on the fact that the Feynman propagator can be written as a sum over all possible paths between the initial and final points in spacetime [11]. In quantum gravity, one should sum over all possible paths linking the two given three-geometries and integrate over all the possible corresponding separations of given local proper times.

From this point of view, R. Balbinot *et al.* [12] proposed a system composed by matter and gravity in which the former follows the latter adiabatically in lattice-points spacetime. It is even more noteworthy that they used the method of Born-Oppenheimer approximation [13]. As is known to us, a wave function acquires the geometric phase in addition to the dynamical one if the system's Hamiltonian modulated by a set of slowly varying external parameter [14]. Towards the matter-gravity (MG) system, we can consider the coordinates associated with the matter as being the fast (light) variables and those associated with the gravity as being the slow (heavy) variables. The traverse along a closed loop C in the external parameter space (superspace) leads to an adiabatic phase which is a matter wave function of the slowly changing heavy degrees of freedom. During the process one passes near a point in which the matter state turns into (double) degeneracy. With regard to the three-dimensional case, a monopole-like

singularity can be used to study the properties of topological structure of the parameter space with the aid of the distribution of Berry curvature. We acquire the Berry curvature by two different way, the path integral method and half-classical approximation, all based on Born-Oppenheimer approximation. We propose a notion of quasi-lattice points in spacetime in the study of half-classical approximation, and it is proved equivalent to the path integral method in calculating Berry curvature. This curvature can be extracted from the linear response of a driven two-level system to nonadiabatic manipulations of its Hamiltonian [15–17].

Recently we have proposed a method of quantum simulation of the magnetic monopoles by use of a driven superconducting qubit [18], and shown that when the monopole charges pass from inside to outside the Hamiltonian manifold, the quantum states influenced by the Berry curvatures ripple in the Hilbert space. In this work, we reconsider the general magnetic force produced by monopole centered at origin of coordinates spanned by a set of parameters in Hamiltonian. In combination with all points of a spatial lattice (all three-space) and spatial metric tensor in spacetime, we compare the singularity in superspace with the magnetic monopole in parameter space.

It is well known that Einstein's theory of general relativity predicts the existence of gravitational waves which are ripples in spacetime [19], created in certain gravitational interactions that travel outward from their sources [20]. The gravitational interactions, like the process of a binary black hole merger, could be viewed as the process of two singularities turning into one. The amplitude of gravitational wave in the merging instant reaches a maximum. The instant corresponds to the moment that ripples emerge in Hilbert space when the monopole travels through (collision) the energy surface of Hamiltonian manifold. As we know, the different number of singularities of physical systems represents the different topological structures [21]. This thus may open a window for the study the geometric and topological properties in quantum gravity with the help of qubits.

In Sec. 2, we introduce a path-integral description of the interaction MG system in the adiabatic approximation. As a gauge connection associated with the Berry curvature is in-

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duced during a closed loop C , we can obtain the nontrivial topological structure in the manifold (a polyhedra) of all points of a spatial lattice (all three-metric). In Sec. 3, We propose a notion of quasi-lattice points in spacetime in the study of half-classical approximation, which is equivalent to the case of path-integral. This method provides the possibility to measure the Berry curvature. As an analogy, in Sec. 4, we utilize an effective method to directly measure the Berry curvature by the way of a nonadiabatic response on physical observables to an external parameter's change rate. Then we set up the connection between the ripples characterized by the fidelity of quantum states in Hilbert space and gravitational waves in microscopic scales. Finally, in Sec. 5, we discuss and summarize the results.

2. EFFECTIVE PATH-INTEGRAL AND QUANTUM GRAVITATIONAL GEOMETRIC TENSOR



FIG. 1: (Color online) Illustration of a spatial lattice. The elementary building blocks for three-dimensional spacetime are simplices of dimension three. A one-simplex is an edge (λ^1 , the blue line), a two-simplex is a triangle (λ^2 , the yellow lines), and a three-simplex is a tetrahedron (λ^3 , the green lines). On a random simplicial lattice there are in general no preferred directions, except for the special path (the maroon part) we choose here.

Now we firstly decompose the manifold of spacetime into

$\Sigma^3 \times \mathbb{R}^1$ where Σ^3 denotes the three-dimensional manifold (spacelike hypersurface x^i , $i = 1, 2, 3$) and \mathbb{R}^1 denotes the real line (time $t = x^0$), and it cannot change upon evolution. The four-dimensional line element under the standard 3+1 decomposition is [1, 4]:

$$ds^2 = -(N^2 - N_i N^i) dt^2 + 2N_i dx^i dt + h_{ij} dx^i dx^j, \quad (2.1)$$

where we use the induced metric $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$, see more details in Appendix A. The total Hamiltonian is given by

$$\begin{aligned} \mathcal{H} &= \mathcal{H}^G + \mathcal{H}^M = \int d^3x (N\mathcal{H}_0^G + N^i \mathcal{H}_i^G) \\ &= \sum_x \lambda^3 [(N\mathcal{H}_0^G + N^i \mathcal{H}_i^G) + (N\mathcal{H}_0^M + N^i \mathcal{H}_i^M)] \\ &\equiv \sum_x \mathcal{H}_x. \end{aligned} \quad (2.2)$$

Here, \mathcal{H}_0^G and \mathcal{H}_i^G are the Hamiltonians associated with gravitational field. \mathcal{H}_0^M and \mathcal{H}_i^M are stress-energy tensor of the matter field projected in the normal direction to the three-dimensional spacelike surface (with one component normal $-N\mathcal{H}_0^M$ and another tangential $-N^i \mathcal{H}_i^M$). The integral over three-space is replaced by a sum over all points x of a spatial lattice of volume λ^3 , thus obtaining a sum of Hamiltonians at every single lattice-point. The lapse function N and shift vector N^i play the role of Lagrange multipliers and satisfy the “proper time” gauge conditions $\partial N / \partial \tau = \partial N^i / \partial \tau = 0$.

To get the wanted gravitational wave function, let us consider the new Hamiltonian in Hilbert space

$$\hat{\mathcal{H}} = \sum_x \lambda^3 [N\hat{\mathcal{H}}_0^R + N^i (\hat{\mathcal{H}}_i^G + \hat{\mathcal{H}}_i^M)] \equiv \sum_x \hat{\mathcal{H}}_x, \quad (2.3)$$

where $\hat{\mathcal{H}}_0^R = \hat{\mathcal{H}}_0^G + \hat{\mathcal{H}}_0^M + \hbar^2 \mathcal{R} / 12$, and \mathcal{R} is the curvature scalar related to the manifold of all three-metrics and has been introduced for the sake of taking into account operator ordering [22].

The MG state vectors of product form $|\phi(h, x), h_{ij}(x)\rangle \equiv |\phi(h, x)\rangle \otimes |h_{ij}(x)\rangle$ where $h_{ij}(x)$ and $\phi(h, x)$ separately describe the gravitational field and the matter field. The transition matrix element (TME) between the initial state $|\phi^\circ, h^\circ\rangle$ and the final state $|\phi^\bullet, h^\bullet\rangle$ can be written as

$$\begin{aligned} \langle \phi^\bullet, h^\bullet | \phi^\circ, h^\circ \rangle &= \int \langle \phi^\bullet, h^\bullet, N^\bullet, N^{\bullet i}, t^\bullet | \phi^\circ, h^\circ, N^\circ, N^{\circ i}, t^\circ \rangle \prod_{x, \tau, i} d[N(t^\bullet - t^\circ)] d[N^i(t^\bullet - t^\circ)] \delta(\partial N / \partial \tau) \delta(\partial N^i / \partial \tau) \\ &= \int \langle \phi^\bullet, h^\bullet | (\exp[-i\epsilon \hat{\mathcal{H}} / \hbar])^N | \phi^\circ, h^\circ \rangle \prod_{x, i} dt^i, \end{aligned} \quad (2.4)$$

where $\epsilon = T/N$ (T denotes the whole evolution time and N is a large number), and $\hat{\mathcal{H}}$ is the Hamiltonian of MG system in

Hilbert space, also the invariant interval between two adjacent lattice-points equals to $N^\bullet t^\bullet - N^\circ t^\circ$ ($N^{\bullet i} t^\bullet - N^{\circ i} t^\circ$). As shown

in Fig. 1, we have divided the interval into N equal segments of size $\lambda(\lambda^i)$ so that $\lambda = (N^\bullet t^\bullet - N^\circ t^\circ)/N$. The “proper time” gauge conditions allow us to introduce a local proper time $t =$

$N(t^\bullet - t^\circ)$ with $N = N^\circ = N^\bullet$ at every single spatial point (similarly for t^i)

$$\begin{aligned} \langle \phi^\bullet, h^\bullet | \phi^\circ, h^\circ \rangle &= \int \langle \phi^\bullet, h^\bullet | \exp[-i\epsilon \hat{\mathcal{H}}(N)/\hbar] | p(N) \rangle \langle p(N) | q(N-1) \rangle \langle q(N-1) | \exp[-i\epsilon \hat{\mathcal{H}}(N-1)/\hbar] \\ &\times \cdots | q(1) \rangle \langle q(1) | \exp[-i\epsilon \hat{\mathcal{H}}(1)/\hbar] | p(1) \rangle \langle p(1) | \phi^\circ, h^\circ \rangle \prod_{x,i,L} dt^i dt \frac{dp_L(N)}{2\pi\hbar} \prod_{k=1}^{N-1} dq^L(k) \frac{dp_L(k)}{2\pi\hbar}, \end{aligned} \quad (2.5)$$

where $\hat{\mathcal{H}}(k)$ describes the Hamiltonian at a lattice-point $k\lambda$ on the evolution path excursing from h° to h^\bullet . We here also use the completeness relation for q^L and its conjugate momentum $p_L = -i\hbar(\delta/\delta q^L)$ [23]. When N tends to infinity, the TME then be given by

$$\langle \phi^\bullet, h^\bullet | \phi^\circ, h^\circ \rangle = \int \exp[iS(h)/\hbar] G(\phi^\bullet, \phi^\circ) \prod_L \frac{1}{2\pi\hbar} \mathcal{D}[p_L] \mathcal{D}[q^L] \prod_{x,i} dt^i dt, \quad (2.6)$$

where the path integral measure can be noted as

$$\int \frac{\mathcal{D}[p_L]}{2\pi\hbar} \mathcal{D}[q^L] = \lim_{n \rightarrow \infty} \prod_{k=1}^{N-1} \left[\int dq^L(k) \right] \prod_{k=1}^N \left[\int \frac{dp_L(k)}{2\pi\hbar} \right], \quad (2.7)$$

and the amplitude for the transition as the matter at a lattice-point $k\lambda$ excursing along the path from h° to h^\bullet can be written as [24, 25]

$$G(\phi^\bullet, \phi^\circ) = \langle \phi^\bullet | \exp[-i\epsilon \hat{\mathcal{H}}^M(N)/\hbar] \cdots \exp[-i\epsilon \hat{\mathcal{H}}^M(k)/\hbar] \cdots \exp[-i\epsilon \hat{\mathcal{H}}^M(1)/\hbar] | \phi^\circ \rangle, \quad (2.8)$$

where $\hat{\mathcal{H}}^M(N)$ is the Hamiltonian of matter, and the gravitational action $S(h)$ reads

$$S(h) = \int d^3x \left[\int_{h^\circ}^{h^\bullet} p_L \delta q^L - \left(\int_{t^\circ}^{t^\bullet} \delta t \hat{\mathcal{H}}_0^R + \int_{t^\circ}^{t^\bullet} \delta t' \hat{\mathcal{H}}_i^G \right) \right]. \quad (2.9)$$

The matter states at every single lattice-point k on the path could be described as follows:

$$G(\phi^\bullet, \phi^\circ) = \sum_{\phi(k=1)}^{\phi(N-1)} \langle \phi^\bullet | \exp[-i\epsilon \hat{\mathcal{H}}^M(N)/\hbar] | \phi(N-1) \rangle \langle \phi(N-1) | \cdots | \phi(1) \rangle \langle \phi(1) | \exp[-i\epsilon \hat{\mathcal{H}}^M(1)/\hbar] | \phi^\circ \rangle, \quad (2.10)$$

where we use the completeness relation of matter states $\sum_{n=1}^N |\phi(n)\rangle \langle \phi(n)| = I$.

Since the mass scale related to gravity (Planck mass, $\sim 10^{-6}g$) is much larger than that related to usual matter, we could suppose the latter follows the former adiabatically. In the Born-Oppenheimer approximation, the matter states might be considered to do the adiabatic motion in the same quantum

number $\phi = \phi^\circ = \phi^\bullet$ [12]. Now we introduce the notion of energy density related to an adiabatic level ϕ at $q^L = q^L(k)$, the Schrödinger equation of matter states could be written as:

$$\hat{\mathcal{H}}_0^M(k) |\phi(k)\rangle = \mathcal{E}^\phi(k) |\phi(k)\rangle, \quad (2.11)$$

then we have

$$G(\phi, \phi) = \exp \left[-\frac{i}{\hbar} \sum_x \sum_k \lambda^3 (\lambda \mathcal{E}^\phi(k) + \lambda^i \langle \phi | \hat{\mathcal{H}}_i^M | \phi \rangle) \right] \lim_{N \rightarrow \infty} \prod_{k=1}^N \langle \phi(k) | \phi(k-1) \rangle. \quad (2.12)$$

We note that each factor $\langle \phi(k) | \phi(k-1) \rangle$ in Eq. (2.12) defines

a connection between two infinitesimally separated points

$\phi(k-1)$ and $\phi(k)$, therefore Eq. (2.12) gives a finite connection along a path given by a set of discrete points $\{k\}$. Thus, by using the Taylor expansion and expanding to the first-order, we obtain

$$\begin{aligned}\langle\phi(k)|\phi(k-1)\rangle &\simeq 1 - \sum_x \lambda^3 \langle\phi(k)|\delta/\delta q^L|\phi(k)\rangle \delta q^L \\ &\approx \exp\left[i \sum_x \lambda^3 \langle\phi(k)|i\delta/\delta q^L|\phi(k)\rangle \delta q^L\right] \\ &= \exp[i\omega/\hbar].\end{aligned}\quad (2.13)$$

From Eq. (2.12), we suppose that

$$\begin{aligned}\lim_{N \rightarrow \infty} \prod_{k=1}^N \langle\phi(k)|\phi(k-1)\rangle &= \langle\phi(t^\bullet)|\phi(t^\circ)\rangle \\ &= \exp[i\Omega_\phi/\hbar]\end{aligned}\quad (2.14)$$

with

$$\Omega_\phi = \int_{h^\circ}^{h^\bullet} \omega = \int_{h^\circ}^{h^\bullet} \delta q^L \langle\phi|i\hbar\delta/\delta q^L|\phi\rangle. \quad (2.15)$$

For simplicity, still in the adiabatic approximation, we let the wave function of matter evolves along a closed loop C in superspace, and $\phi(0) = \phi(t^\circ)$, $\phi(T) = \phi(t^\bullet)$, then Eq. (2.15) turns into

$$\Omega_\phi(C) = \oint_C \omega = \oint_C \delta q^L \langle\phi|i\hbar\delta/\delta q^L|\phi\rangle. \quad (2.16)$$

This form is essentially the same as Berry phase (actually, this is a variational version of Berry phase). Therefore, we obtain the effective transition amplitude related to the adiabatic change of the external dynamical variable ϕ ,

$$G_{\text{eff}}(T) = \exp\left[-\frac{i}{\hbar} \int d^3x \left(\int_0^T \delta t \langle\phi|\hat{\mathcal{H}}_0^{\text{M}}|\phi\rangle + \int_0^{T'} \delta t' \langle\phi|\hat{\mathcal{H}}_i^{\text{M}}|\phi\rangle - \oint_C \delta q^L \langle\phi|i\hbar\delta/\delta q^L|\phi\rangle \right)\right]. \quad (2.17)$$

The effective path-integral for the TME is given by

$$\langle\phi^\bullet, h^\bullet|\phi^\circ, h^\circ\rangle_{\text{eff}} = \int \exp\left[\frac{i}{\hbar} \left(S_{ad} + \int d^3x \int_{h^\circ}^{h^\bullet} \delta q^L \langle\phi|i\hbar\delta/\delta q^L|\phi\rangle \right)\right] \prod_L \frac{1}{2\pi\hbar} \mathcal{D}[p_L] \mathcal{D}[q^L] \prod_{x,i} dt^i dt \quad (2.18)$$

with the adiabatic action function [25]

$$S_{ad} = S(h) - \int d^3x \left(\int_{t^\circ}^{t^\bullet} \delta t \mathcal{E}^\phi + \int_{t^\circ}^{t^\bullet} \delta t^i \langle\phi|\hat{H}_i^{\text{M}}|\phi\rangle \right). \quad (2.19)$$

The phase $\Omega_\phi(C)$ appears as a topological action function in MG system. This implies that the eigenstate of matter which is only the function of the three-metric q^L stays the same during the motion of the system in superspace. In the presence of an effective “gauge” field which can be described by the “vector potential” $\mathcal{A} = \langle\phi|i\hbar\delta/\delta q^L|\phi\rangle$, one might have nontrivial topological structure in the manifold of superspace in case one allows for a gauge transformation of it. The connection \mathcal{A} is substantially the Berry-Simon connection [26].

According to adiabatic condition, the matter states always stay in the same quantum number in Eq. (2.12). The lowest correction to this will allow one to jump to another eigenstate and then back to the initial eigenstate. Thus the TME, also called quantum gravitational geometric tensor, reads

$$\begin{aligned}\mathcal{G}_{XY}^\phi &= \sum_{\phi' \neq \phi} \langle\phi|\frac{\delta}{\delta q^X}|\phi'\rangle \langle\phi'|\frac{\delta}{\delta q^Y}|\phi\rangle \\ &= - \sum_{\phi' \neq \phi} \frac{1}{(\mathcal{E}^\phi - \mathcal{E}^{\phi'})^2} \langle\phi|\frac{\delta \hat{\mathcal{H}}_0^{\text{M}}}{\delta q^X}|\phi'\rangle \langle\phi'|\frac{\delta \hat{\mathcal{H}}_0^{\text{M}}}{\delta q^Y}|\phi\rangle,\end{aligned}\quad (2.20)$$

where \mathcal{G}_{XY}^ϕ is Hermitian and we note that the eigenfunction of

the matter acquires an opposite adiabatic phase to the gravitational wave function. The TME in Eq. (2.20) could be decomposed into real symmetric and imaginary antisymmetric parts [27]:

$$\text{Re}\mathcal{G}_{XY}^\phi = \frac{1}{2}(\mathcal{B}_{XY}^\phi + \mathcal{B}_{YX}^\phi), \quad \text{Im}\mathcal{G}_{XY}^\phi = \frac{1}{2}(\mathcal{B}_{XY}^\phi - \mathcal{B}_{YX}^\phi). \quad (2.21)$$

The real part in Eq. (2.21) provides a way to measure the distance between two neighbouring matter wave functions along paths in superspace (parameter space), and the imaginary part can be written as

$$\begin{aligned}\text{Im}\mathcal{G}_{XY}^\phi &= \frac{1}{2} \left(\frac{\delta}{\delta q^X} \langle\phi|\frac{\delta}{\delta q^Y}|\phi\rangle - \frac{\delta}{\delta q^Y} \langle\phi|\frac{\delta}{\delta q^X}|\phi\rangle \right) \\ &= - \sum_{\phi' \neq \phi} \frac{1}{(\mathcal{E}^\phi - \mathcal{E}^{\phi'})^2} \left[\langle\phi|\frac{\delta \hat{\mathcal{H}}_0^{\text{M}}}{\delta q^X}|\phi'\rangle \right. \\ &\quad \times \left. \langle\phi'|\frac{\delta \hat{\mathcal{H}}_0^{\text{M}}}{\delta q^Y}|\phi\rangle - (q^Y \leftrightarrow q^X) \right],\end{aligned}\quad (2.22)$$

which is a phase 2-form in superspace, also called the gravitational Berry curvature. It indicates that degeneracies are

some singularities (mini-black holes, might be) that will contribute nonzero terms to topological invariants [15, 16]. From

$$\int d^3x \oint_C \delta q^L \langle \phi | i\hbar \delta / \delta q^L | \phi \rangle = - \int d^3x \iint_\sigma \delta \sigma^{XY} \text{Im} \sum_{\phi' \neq \phi} \frac{\langle \phi | \delta \hat{\mathcal{H}}_0^M / \delta q^X | \phi' \rangle \langle \phi' | \delta \hat{\mathcal{H}}_0^M / \delta q^Y | \phi \rangle}{(\mathcal{E}^\phi - \mathcal{E}^{\phi'})^2}, \quad (2.23)$$

where the summation in superspace are over the three-geometries on the spacelike three-surface and $\delta \sigma^{XY}$ is the element of a two-sphere in superspace bounded by the loop C . As shown in Fig. 2, the gravitational Berry phase under the background of a continuous spatial lattice form, (the transition from a smooth triangulation of a sphere to the corresponding secant approximation).

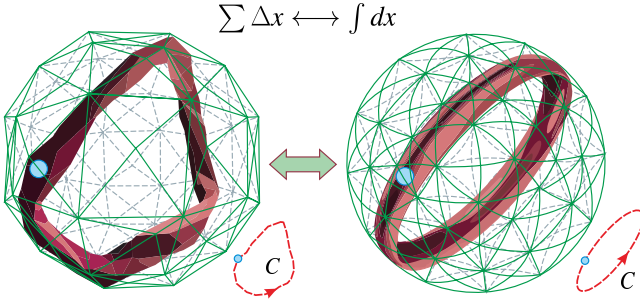


FIG. 2: (Color online) Polyhedral approximation to a sphere. The green spatial lattice in the left is a closed three-dimensional manifold with all three-space (sum over all points of a spatial lattice) which is homeomorphic to a connected polyhedron in piecewise linear space, and its points possess neighborhoods which are homeomorphic to the interior of the three-dimensional sphere (in the right). According to Gauss-Bonnet theorem [28], the topological structure between the polyhedra and the sphere is equivalent to each other (both have the same Euler characteristic). The light blue ball represents the matter states associated with the closed red trajectories C of an adiabatic evolution. The dashed red line is the simplification of the red cyclic polyhedron (all possible paths with the same quantum number corresponding to a net at each time, not a point) which is homeomorphic to the red torus in the right.

3. HALF-CLASSICAL ADIABATIC REACTIONS OF MATTER-GRAVITY SYSTEM

In the Born-Oppenheimer researches of molecules, the dynamics of a composite physical system could be divided into two parts: fast (light) system, which is described by fast variables (e.g., the position \mathbf{q} and momenta \mathbf{p} of the electrons); and slow (heavy) system, which is described by slow variables (e.g., the position \mathbf{Q} and momenta \mathbf{P} of the nuclei).

In MG system, we could consider the coordinates associated with the matter as being the light variables and those associated with the gravity as being the heavy variables. Thus

Eq. (2.12) to Eq. (2.17), the gravitational Berry phase can be given by

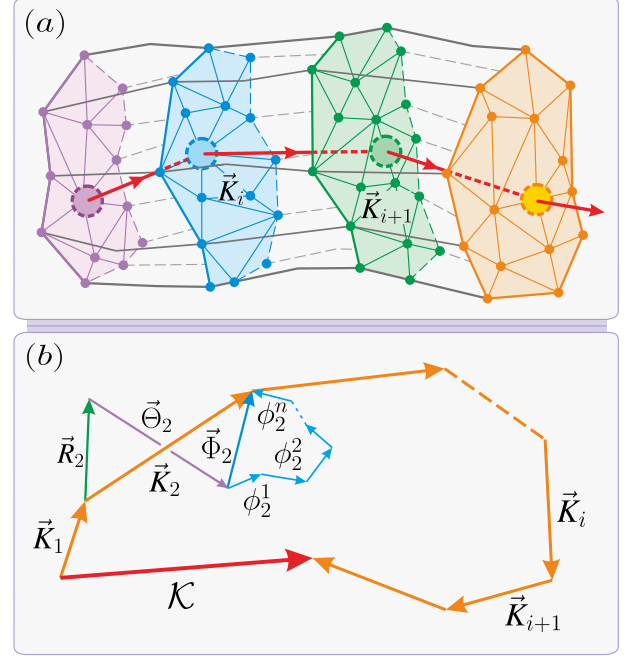


FIG. 3: (Color online) Quasi-lattice points in spacetime. (a) The trajectory of lattice-points depicts the evolution history of matter state in gravitational field. The simultaneously created lattice-points constitute a spatial net at every moment. Points on every spatial net can be viewed as equivalent to a quasi-lattice point on the trajectory of evolution (dashed circles). So the position of an ensemble of lattice-points could be described by a set of quasi-lattice vectors. The quasi-lattice point from one to another reads \vec{K}_i to \vec{K}_{i+1} . (b) Quasi-lattice vector function \mathcal{K} in vector space. The function \mathcal{K} can be viewed as a set of quasi-lattice vectors $\{\{\vec{K}_1\}, \dots, \{\vec{K}_n\}\}$, and a quasi-lattice vector $\{\vec{K}_n\}$ can be represented by a set of feature vectors $\{\vec{X}_n, \vec{Y}_n, \vec{Z}_n, \dots\}$, and a feature vector \vec{X}_n is composed of a set of feature components $\{x_1, \dots, x_n\}$. For example, here we set $\vec{X}_n = \vec{R}_n, \vec{Y}_n = \vec{\Theta}_n, \vec{Z}_n = \vec{\Phi}_n$, and $x_n^n = r_n^n, y_n^n = \theta_n^n, z_n^n = \phi_n^n$, (here the subscript $n = 2$). This example also reflects the spacetime is emergent which is not a fundamental physical quantity.

we define the position of a single quasi-lattice point which could be considered as a set of quasi-lattice vectors of spacetime, as shown in Fig. 3, the quasi-lattice vector function is given by

$$\mathcal{K} = \ddagger \{ \{ \vec{K}_i \} \} \ddagger = \ddagger \{ \{ \vec{K}_1 \}, \dots, \{ \vec{K}_n \} \} \ddagger$$

$$\begin{aligned}
&= \ddagger \{ \{ \vec{X}_1, \vec{Y}_1, \vec{Z}_1, \dots \}, \dots, \{ \vec{X}_n, \vec{Y}_n, \vec{Z}_n, \dots \} \} \ddagger \\
&= \{ \{ \vec{X}_1, \dots, \vec{X}_n \}, \{ \vec{Y}_1, \dots, \vec{Y}_n \}, \{ \vec{Z}_1, \dots, \vec{Z}_n \}, \dots \} \\
&= \{ \{ \vec{K}^1 \}, \dots, \{ \vec{K}^n \} \} = \{ \{ \vec{K}^i \} \}, \quad (3.1)
\end{aligned}$$

where the operator $\ddagger \{ \{ \} \} \ddagger$ is called the normal emergent set, that also can be transformed into the corresponding emergent set $\{ \{ \} \}$.

Now let the evolution of matter state be described by a density matrix $\hat{\rho}(t)$ driven by the Hamiltonian $\hat{\mathcal{H}}_0^{\text{M}}$, which is time-dependent because the quasi-lattice point \vec{K}_i changes with time. As is known to us, there appear magnetic reaction force at the first order, associated with the geometric phase [29]. Then we assume that the gravitational reaction has the same expression. The evolution of ρ is governed by [30]

$$i\hbar\epsilon\dot{\hat{\rho}}(t) = [\hat{\mathcal{H}}_0^{\text{M}}(\mathcal{K}(t)), \hat{\rho}(t)], \quad \text{Tr}\hat{\rho} = 1, \quad (3.2)$$

where ϵ is the adiabatic parameter and $\mathcal{K}(t) \propto t$ implies $q^L(\mathcal{K}) \propto t$. The desired force is given by

$$\vec{F} = -\text{Tr}\hat{\rho} \nabla \hat{\mathcal{H}}_0^{\text{M}}. \quad (3.3)$$

One may write $\hat{\rho}$ as the series in powers of ϵ

$$\hat{\rho} \equiv \sum_{r=0}^{\infty} \epsilon^r \hat{\rho}_r. \quad (3.4)$$

The terms $\hat{\rho}_r$ are determined by the following equations:

$$[\hat{\mathcal{H}}_0^{\text{M}}, \hat{\rho}_0] = 0, \quad [\hat{\mathcal{H}}_0^{\text{M}}, \hat{\rho}_r] = i\hbar\dot{\hat{\rho}}_{r-1} \quad (r > 0). \quad (3.5)$$

We can choose $\hat{\rho}_0$ as one of the pure matter states as in Eq. (2.11), we redefine the adiabatic eigenstates and the energy levels by

$$\hat{\mathcal{H}}_0^{\text{M}}(\mathcal{K}(t))|\phi^m(\mathcal{K}(t))\rangle = \mathcal{E}^\phi(\mathcal{K}(t))|\phi^m(\mathcal{K}(t))\rangle, \quad (3.6)$$

say, the ϕ^n -th is

$$\hat{\rho}_0(t) = |\phi^n(\mathcal{K}(t))\rangle\langle\phi^n(\mathcal{K}(t))|. \quad (3.7)$$

Eq. (3.7) depends on time through varying the position of each quasi-lattice vector function $\mathcal{K}(t)$.

Now we can write the general force (3.3) as

$$\begin{aligned}
\vec{F} &= -\text{Tr}\hat{\rho}_0 \nabla \hat{\mathcal{H}}_0^{\text{M}} - \epsilon \text{Tr}\hat{\rho}_1 \nabla \hat{\mathcal{H}}_0^{\text{M}} + \mathcal{O}(\epsilon^2) \\
&= -\nabla \mathcal{E}^{\phi^n}(\mathcal{K}) + \epsilon \vec{F}_1 + \mathcal{O}(\epsilon^2), \quad (3.8)
\end{aligned}$$

where

$$\vec{F}_1 \equiv -\text{Tr}\hat{\rho}_1 \nabla \hat{\mathcal{H}}_0^{\text{M}} = -\sum_{\phi^k, \phi^l} \langle \phi^k | \hat{\rho}_1 | \phi^l \rangle \langle \phi^l | \nabla \hat{\mathcal{H}}_0^{\text{M}} | \phi^k \rangle, \quad (3.9)$$

the leading term $-\nabla \mathcal{E}^{\phi^n}$ (also equal to $-\langle \phi^n | \nabla \hat{\mathcal{H}}_0^{\text{M}} | \phi^n \rangle$) in Eq. (3.8) is the Born-Oppenheimer force, and the second term is the desired first-order reaction. Note that $\hat{\rho}_1(\vec{v})$ is a function of slow velocity $\vec{v} \equiv \partial \mathcal{K}(t)/\partial t$.

Under the basis of adiabaticity, the off-diagonal elements of the corrections $\hat{\rho}_r$ are determined by the Eq. (3.5) as

$$\langle \phi^k | \hat{\rho}_r | \phi^l \rangle = i\hbar \frac{\langle \phi^k | \dot{\hat{\rho}}_{r-1} | \phi^l \rangle}{\mathcal{E}^{\phi^k} - \mathcal{E}^{\phi^l}}, \quad (\phi^k \neq \phi^l). \quad (3.10)$$

The diagonal elements are settled in the pure state condition $\hat{\rho}(t) = \hat{\rho}(t)^2 = |\varphi(t)\rangle\langle\varphi(t)|$. Associated with Eq. (3.4), we obtain the first-order force as

$$\vec{F}_1 = -i\hbar\vec{v} \cdot \sum_{\phi^k, \phi^l} \langle \phi^k | \nabla \phi^l \rangle (\delta_{\phi^n \phi^l} - \delta_{\phi^n \phi^k}) \langle \phi^l | \nabla \phi^k \rangle = i\hbar\vec{v} \wedge \sum_{\phi^k} \langle \nabla \phi^n | \phi^k \rangle \wedge \langle \phi^k | \nabla \phi^n \rangle, \quad (3.11)$$

where we use $\langle \phi^l | \nabla \hat{\mathcal{H}}_0^{\text{M}} | \phi^k \rangle / (\mathcal{E}^{\phi^k} - \mathcal{E}^{\phi^l}) = \langle \phi^l | \nabla \phi^k \rangle$, $(\phi^k \neq \phi^l)$ and completeness relation.

This expression has the form which is analogues to Lorentz force $\vec{F}_1 = \vec{v} \wedge \vec{G}(\mathcal{K})$, where the “gravitational field” is ($\hbar \equiv 1$ as follows)

$$\begin{aligned}
\vec{G}(\mathcal{K}) &= -\text{Im} \langle \nabla \phi^n(\mathcal{K}) | \wedge | \nabla \phi^n(\mathcal{K}) \rangle \\
&= -\text{Im} \sum_{\phi^m \neq \phi^n} \frac{\langle \phi^n(\mathcal{K}) | \nabla_{\vec{K}^m} \hat{\mathcal{H}}_0^{\text{M}}(\mathcal{K}) | \phi^m(\mathcal{K}) \rangle \langle \phi^m(\mathcal{K}) | \nabla_{\vec{K}^n} \hat{\mathcal{H}}_0^{\text{M}}(\mathcal{K}) | \phi^n(\mathcal{K}) \rangle - (\vec{K}^n \leftrightarrow \vec{K}^m)}{(\mathcal{E}^{\phi^m}(\mathcal{K}) - \mathcal{E}^{\phi^n}(\mathcal{K}))^2}, \quad (3.12)
\end{aligned}$$

which is similar with the symplectic 2-form resulting the ge-

ometric phase in the MG system when \mathcal{K} is cycled for the

gravitational field. For a sphere of lattice-points, we set \mathcal{K}_S to be the quasi-lattice vector function of an emergent set of quasi-lattice vectors $\{\{\vec{K}_S^1\}, \{\vec{K}_S^2\}, \{\vec{K}_S^3\}\}$, where $\{\vec{K}_S^1\} = \{\vec{R}^1, \dots, \vec{R}^n\}$, $\{\vec{K}_S^2\} = \{\vec{\Theta}^1, \dots, \vec{\Theta}^n\}$, $\{\vec{K}_S^3\} = \{\vec{\Phi}^1, \dots, \vec{\Phi}^n\}$. To ensure there have only two parameter sets of the Hamiltonian ($\vec{\Theta}^n$ and $\vec{\Phi}^n$), we assume that the average module of radius of the lattice sphere away from the origin during the process of the evolution of the matter state reads

$$\|\langle \vec{R}^n \rangle\|_0^T = \text{const}, \quad \text{or} \quad \frac{d}{dt} \langle \|\vec{R}^n\| \rangle_0^T = 0. \quad (3.13)$$

After this handle, the closed evolution trajectory of the matter wave function on a two-dimensional sphere in superspace can be vividly pictured by Fig. 2. The structure of the first-order force exemplifies linear response theory, with the force $\epsilon \vec{F}_1$ and the slow velocity \vec{v} being related by the antisymmetric tensor which is the very gravitational Berry curvature in Eq. (2.22). Thus we may conclude that the theory of half-classical approximation which introduces the notion of quasi-lattice point in superspace is equivalent to the case of path-integral.

4. RIPPLES IN HILBERT SPACE AND GRAVITATIONAL WAVES IN SUPERSPACE – PHYSICAL QUANTUM SIMULATION

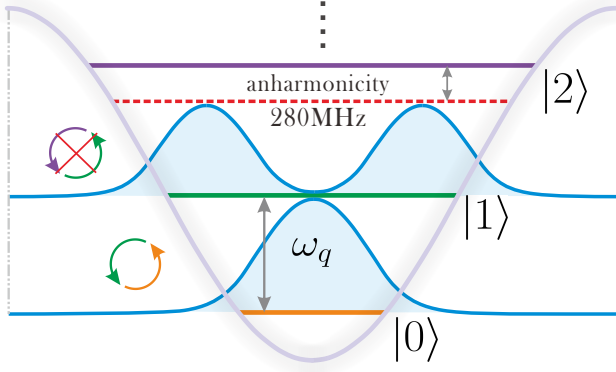


FIG. 4: (Color online) Energy spectrum of the transmon qubit. Here we assume the qubit which with a transition frequency of $\omega_q = 4.395$ GHz is effectively a nonlinear resonator, and the anharmonicity of 280 MHz, ensures that the qubit transition only occurs between the two lowest energy levels [31].

A recent work studies with quantum simulation of the magnetic monopoles by a driven superconducting qubit [18], and shows when the monopole charges pass from inside to outside the Hamiltonian manifold, the quantum states influenced by the Berry curvatures will ripple in the Hilbert space. In order to make a comparison between the Berry curvature in the Hilbert space and the curvature of the spacetime in the superspace (the gravitational Berry curvature as mentioned before). We firstly consider a driven superconducting transmon qubit

with an anharmonicity of 280 MHz forming an effective two-level system, as seen in Fig. 4. The Hamiltonian of the system can be written as

$$\hat{H} = \frac{1}{2} \begin{pmatrix} \Delta & \Omega e^{-i\phi} \\ \Omega e^{i\phi} & -\Delta \end{pmatrix}, \quad (4.1)$$

where the detunings $\Delta = \omega_m - \omega_q = \Delta_1 \cos \theta + \Delta_2$ (ω_m is the microwave driven frequency and ω_q is the qubit transition frequency), and the Rabi frequency $\Omega = \Omega_1 \sin \theta$. ϕ is the phase of drive tone. By changing Δ_1, Δ_2 and Ω_1 , we can acquire the desired single-qubit Hamiltonian that can be described in an ellipsoidal manifold spanned by these parameters. Here, we set ellipsoids of size $\Delta_1 = 6\pi \times 10$ MHz and $\Omega = 3\pi \times 10$ MHz, and manipulate Δ_2 from 0 to $2\Delta_1$. It is worth noting that if $\Delta = \Omega = 0$, this corresponds to a degeneracy in parameter space which can be viewed as a magnetic monopole [18]. This also can be viewed as that the geometric magnetic force is that of a monopole of strength that is dependent on the position vector centered at $\Delta = \Omega = 0$.

In order to establish an association between monopole in parameter space and singularity in superspace. We need to introduce the method to measure the Berry curvature first. In this case, the Berry curvature is given by

$$F_{\theta\phi} = i \frac{\langle 0 | \partial_\theta \hat{H} | 1 \rangle \langle 1 | \partial_\phi \hat{H} | 0 \rangle - \langle 0 | \partial_\phi \hat{H} | 1 \rangle \langle 1 | \partial_\theta \hat{H} | 0 \rangle}{(E_1 - E_0)^2}, \quad (4.2)$$

where E_n and $|n\rangle$ are the n -th eigenvalue ($n = 0, 1$) and their corresponding eigenstates of the Hamiltonian \hat{H} , respectively. Ref. [15] states that the local Berry curvature can be extracted from the linear response of the qubit during a nonadiabatic passage. This idea is, the motion of the quantum states in a curved space will be deviated from a straight trajectory in flat space. Thus the Berry curvature can be calculated from the deflection from adiabaticity [17]. If we manipulate this qubit by controlling a set of parameters ($\Delta_1, \Delta_2, \Omega_1$) of its Hamiltonian with the rate of change of a parameter, then the state of the system feels a geometric (magnetism, gravity, etc.) force $F_\phi \equiv -\langle \psi(t) | \partial_\phi \hat{H} | \psi(t) \rangle$, given by

$$F_\phi = \text{const} + \theta_t F_{\theta\phi} + \mathcal{O}(\theta_t^2), \quad (4.3)$$

where θ_t is the rate of change of the parameter θ (quench velocity) and $F_{\theta\phi}$ is a component of the Berry curvature tensor. This is analogous to the case under the condition when parameters of vectors are $\vec{K}^m = \vec{K}_S^2$ and $\vec{K}^n = \vec{K}_S^3$ in Eq. (3.12). To neglect the higher-order nonlinear term $\mathcal{O}(\theta_t^2)$, the system parameters should be ramped slowly enough, see more details in Appendix B. Noting that Eq. (4.3) is a specific form of Eq. (3.8) in the case of magnetism originated from monopole in parameter space. Analogously, gravity generated by singularity in superspace can make the spacetime (spatial lattice) curve, and the curving degree of the superspace also can be described by the gravitational Berry curvature in Eq. (3.12).

Therefore, we reconsider the general magnetic force produced by monopole centered at origin of coordinates spanned by a set of parameters in Hamiltonian. Associating with all

three-space and spatial metric tensor in spacetime, we compare the singularity in superspace with the magnetic monopole in parameter space. Einstein's theory of general relativity predicts the existence of gravitational waves which are ripples in spacetime, created in certain gravitational interactions that travel outward from their sources. Then we establish the relationship between the ripples characterized by the fidelity of quantum superposition state in Hilbert space and gravitational waves in microscopic scales (the mass scale of the matter is much less than the Planck mass).

From Fig. 5, we note that the number of singularity changes. In Fig. 5 (a), every mini-black hole corresponds to a singularity, after the merger of two mini-black holes, the number of singularity of the binary system jumps from two to one. In Fig. 5 (b), after the monopole traversing outside the manifold of the parameter space, the number of singularity of the two-level system jumps from one to zero. These processes reflect the change of topological structures of both systems. The amplitude of gravitational wave from the merger of two black holes reaches a maximum. This instant corresponds to the moment that ripples are generated in Hilbert space when the monopole travels through ($\Delta_2/\Delta_1 = 1$) the energy surface of Hamiltonian manifold. In Fig. 5 (b), the fidelity of the target superposition state $|\psi\rangle = 1/\sqrt{2}(|g\rangle + |e\rangle)$ is plotted versus Δ_2/Δ_1 at $\theta = \pi$, where the fidelity of the superposition state is defined as $f = \langle\psi|\hat{\rho}(T)|\psi\rangle$. We also note that the fidelity of the superposition state is slightly oscillate around 0.5 when at the region of $|\Delta_2/\Delta_1| < 0.5$ and $|\Delta_2/\Delta_1| > 1.5$. This implies the quantum states almost stay in the eigenstates ($|e\rangle$ or $|g\rangle$) at these two regions.

As shown in Fig. 5, we note that from $|\Delta_2/\Delta_1| = 0$ to $|\Delta_2/\Delta_1| = 1$, the ripples in Hilbert space affected by the parameter space (energy space) reflect the change of quantum states from the eigenstates (two matter states) to the superposition state. The waveform in the region of $|\Delta_2/\Delta_1| = 1$ reflects the monopole travelling through ($\Delta_2/\Delta_1 = 1$) the energy surface of Hamiltonian manifold, this could be viewed as the creation of gravitational fields by the merger of two mini-black holes. When $|\Delta_2/\Delta_1| > 1$, this afterwind could be viewed as affected by the aftershock in parameter space.

Notice in Ref. [18] that, during the process of topological transition, the fidelity of the quantum states is affected by the Berry curvature, that can be expressed by

$$F_{\theta\phi} = \frac{\langle\partial_\phi\hat{H}\rangle}{\theta_t} = \frac{\Omega_n \sin\theta}{2\theta_t} \langle\psi|\hat{\sigma}_y|\psi\rangle, \quad (4.4)$$

where $|\psi\rangle$ is the superposition state. As is known to us, the strength of gravitational field can be represented by the curvature of the structure of spacetime. The fidelity of the quantum eigenstates is affected by the strength of the magnetic fields which are emitted from the monopoles. Then the interaction between matter states and gravity could be described by the Berry-like curvature (gravitational field) which reflects the changes of the curvature of spacetime in superspace.

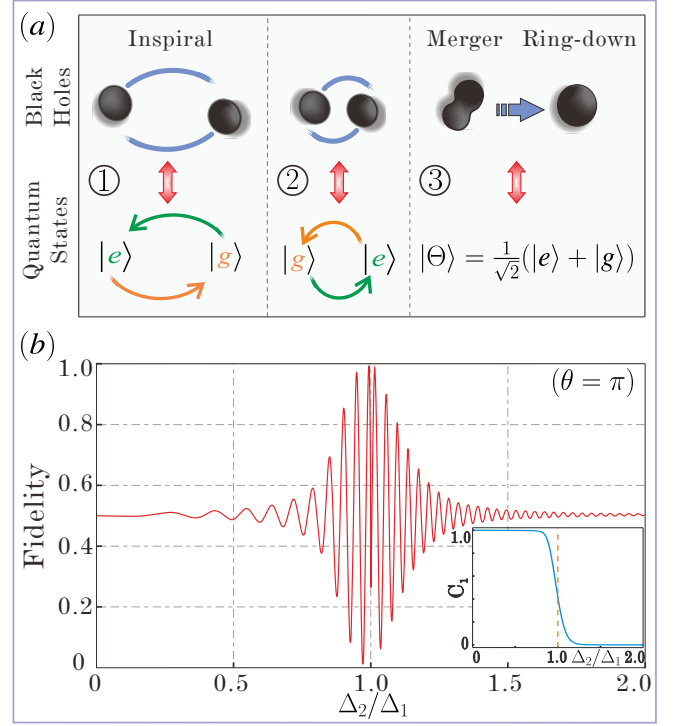


FIG. 5: (Color online) Quantum simulation of microscopic gravitational waves with a two-level system. (a) The gravitational waves in microscopic scales that emitted by the inspiral and merger of two mini-black holes correspond to the process of population inversion of the two eigenstates and the process of the transfer into the superposition state, respectively. The frequency of the inspiral of mini-black holes is analogous to the frequency of the population inversion of the two eigenstates. (b) Ripples of the wave function of superposition state in Hilbert space. Here we set $\Delta_1 = 6\pi \times 10$ MHz and $\Omega = 3\pi \times 10$ MHz, and manipulate Δ_2 from 0 to $2\Delta_1$ at $\theta = \pi$. The fidelity oscillates faster and faster with the increasing of Δ_2/Δ_1 before the topological transition [18] (the first Chern number C_1 turns from 1 to 0 at Δ_2/Δ_1), corresponding to the period of inspiral of the two mini-black holes becomes shorter and shorter.

5. CONCLUSION

Based on the approach of adiabatic approximation, we investigate the geometrical and topological structure which are described by the gravitational Berry curvature under the theory of quantum gravitation by applying the path integral method and half-classical approximation. We show the theory of half-classical approximation which introduces the notion of quasi-lattice point in superspace is equivalent to the case of path-integral. Offering the notion that quasi-lattice vector function also reflects the spacetime is emergent, which is not a fundamental physical quantity.

An artificial magnetic monopole formed in parameter space of the Hamiltonian of a driven two-level system (by a superconducting qubit in this work) gives rise to ripples (characterized here by the fidelity of quantum states) in Hilbert space when it travels through the surface of energy manifold spanned by system parameters. The magnetic field

can be represented by Berry curvature, and the distribution of Berry curvature can reflect both the shape of the manifold in parameter space of the system's Hamiltonian and the population of quantum states. Thus the motion of the quantum states in the curved parameter space can be utilized to stimulate the evolution of matter state in the curved superspace. The first-order reaction force, also called general force, can be viewed as a characterization of the degree of Berry curvature. For example, the general magnetism force originated from monopole in superspace is analogous to the general gravitational force originated from singularity in superspace, the corresponding Berry curvatures reflects the strength of the magnetic field produced by the monopole and the strength of the gravitational field produced by the singularity (can be viewed as a mini-black hole).

On the other hand, as we know, the different number of singularities of physical systems indicates the different topological structure [21]. Thus the method of measuring the topological change in both systems may open a window for the study of the geometric and topological properties in quantum gravity with the help of qubits. This may open up a new pathway to explore the geometrical and topological properties of quantum gravity with the aid of qubit systems.

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Appendix A: Proof of Eq. (2.1)

Firstly, we introduce a spacelike hypersurface into the manifold of spacetime $X^\mu = X^\mu(t, x^i)$, $\mu = 0, 1, 2, 3$ and give the normal vector n^μ and the tangent vector $X_i^\mu \equiv \partial X^\mu / \partial x^i$ to any point on it. The local framework of four-dimensional (n^μ, X_i^μ) satisfies the following three conditions:

- (1) orthogonality: $g_{\mu\nu} X_i^\mu n^\nu = 0$;
- (2) metric on spacelike hypersurface: $h_{ij} = g_{\mu\nu} X_i^\mu X_j^\nu$;
- (3) timelike: $g_{\mu\nu} n^\mu n^\nu = -1$.

For spacelike hypersurface, we let $i, j = 1, 2, 3$. As shown in Fig. 6, continuous deformation of the hypersurface in spacetime could be assumed. The deformation vector N^μ can be defined as $N^\mu \equiv \partial X^\mu(t, x^i) / \partial t$. In terms of a normal to the surface n^μ , one has

$$N^\mu = N n^\mu + N^i X_i^\mu. \quad (\text{A1})$$

In Fig. 6, the spacetime interval of the deformation of the hypersurface is given by

$$\begin{aligned} ds^2 &= g_{\mu\nu} dX^\mu dX^\nu = g_{\mu\nu} N^\mu N^\nu dt dt + 2g_{\mu\nu} N^\mu X_i^\nu dt dx^i + g_{\mu\nu} X_i^\mu X_j^\nu dx^i dx^j \\ &= g_{tt} dt dt + 2g_{ti} dt dx^i + g_{ij} dx^i dx^j, \end{aligned} \quad (\text{A2})$$

where g_{ij} ($i, j = 1, 2, 3$) denotes the three-metric on the given spacelike hypersurface. Therefore

$$\begin{aligned} g_{ij} &= X_i^\mu X_j^\nu g_{\mu\nu} = X_i^\mu X_j^\nu (h_{\mu\nu} - n_\mu n_\nu) = X_i^\mu X_j^\nu h_{\mu\nu} = h_{ij}, \\ g_{ti} &= N^\mu X_i^\nu g_{\mu\nu} = g_{\mu\nu} (N n^\mu + N^j X_j^\mu) X_i^\nu = N^j g_{\mu\nu} X_j^\mu X_i^\nu = N^j h_{ij} = N_i, \\ g_{tt} &= N^\mu N^\nu g_{\mu\nu} = N^\mu N^\nu (h_{\mu\nu} - n_\mu n_\nu) = (N n^\mu + N^i X_i^\mu) (N n^\nu + N^j X_j^\nu) h_{\mu\nu} - N^2 \\ &= N^i X_i^\mu N^j X_j^\nu h_{\mu\nu} - N^2 = N^i N_i - N^2. \end{aligned} \quad (\text{A3})$$

This is the proof of Eq. (2.1).

Appendix B: Proof of Eq. (4.3)

Now we consider a system under the domination of the Hamiltonian $\hat{H}(t) = \hat{H}_0 + v(t)\hat{U}$, where \hat{H}_0 is stationary and $v(t)\hat{U}$ is time-dependent [32]. In order to characterize

the dynamics of the system resulting from the time-dependent perturbation theory, we assume that $v(t)$ is a linear function of time

$$v(t) = v_i + t v_t (v_f - v_i), \quad 0 \leq t \leq 1/v_t. \quad (\text{B1})$$

Here v_t is the rate of change of the parameter $v(t)$ and $v_t \rightarrow 0$ denotes the adiabatic limit, while $v_t \rightarrow \infty$ denotes the sudden quench. The values of the vectors v_i and v_f can be ar-

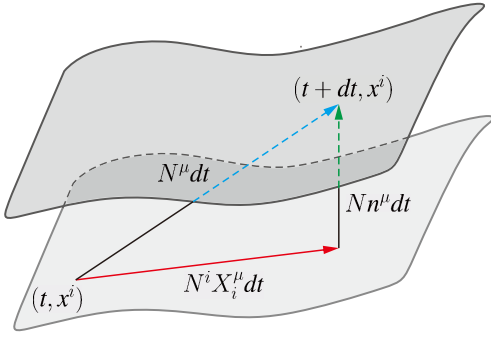


FIG. 6: (Color online) Illustration of the lapse function and the shift vector. These two quantities depict the lapse of proper time (N) between two infinitesimally close hypersurfaces, and the corresponding shift in spatial coordinate (N^i).

bitrarily far from each other in principle. Let us denote the instantaneous eigenstates of the Hamiltonian $\hat{H}(t)$ as $|n\rangle$ satisfies the equation $\hat{H}(t)|n\rangle = E_n(t)|n\rangle$, where $E_n(t)$ is the corresponding instantaneous eigenvalue. We express the wave function in terms of the instantaneous eigenstates, $|\psi(t)\rangle = \sum_n a_n(t) e^{-i\theta_n(t)} |n\rangle$, where $\theta_n(t) = \int_{t_i}^t E_n(\tau) d\tau$ ($\hbar \equiv 1$), and the lower limit of integration in the expression for $\theta_n(t)$ is arbitrary. Then, the Schrödinger equation reads

$$\partial_t a_n(t) = - \sum_m a_m(t) \langle n | \partial_t | m \rangle e^{i(\theta_n(t) - \theta_m(t))}, \quad (\text{B2})$$

which can also be rewritten as an integral equation

$$a_n(t) = - \int_{t_i}^t dt' \sum_m a_m(t') \langle n | \partial_{t'} | m \rangle e^{i(\theta_n(t') - \theta_m(t'))}. \quad (\text{B3})$$

If the energy levels $E_n(\tau)$ and $E_m(\tau)$ are not generate, the matrix element $\langle n | \partial_t | m \rangle$ can be written as

$$\langle n | \partial_t | m \rangle = - \frac{\langle n | \partial_t \hat{H} | m \rangle}{E_n(t) - E_m(t)} = - \frac{v_t \langle n | \hat{U} | m \rangle}{E_n(t) - E_m(t)}. \quad (\text{B4})$$

If $v(t)$ is a monotonic function of time t then in Eq. (B3) one can manipulate variables from t to $v(t)$ and obtain

$$a_n(v) = - \int_{v_i}^v dv' \sum_m a_m(v') \langle n | \partial_{v'} | m \rangle e^{i(\theta_n(v') - \theta_m(v'))}, \quad (\text{B5})$$

where

$$\theta_n(v) = \int_{v_i}^v dv' \frac{E_n(v')}{v'_t}. \quad (\text{B6})$$

Eqs. (B3) and (B5) allow for a systematic expansion of the solution in the small parameter v_t . Indeed, in the limit $v_t \rightarrow 0$ all the transition probabilities are suppressed because the phase factors are strongly oscillating functions of v . In the leading order in v_t only the term with $m = n$ should be retained in the sums in Eqs. (B3) and (B5). It results in the Berry phase

$$\gamma_n(t) = -i \int_{t_i}^t dt' \langle n | \partial_{t'} | n \rangle = -i \int_{v_i}^{v(t)} dv' \langle n | \partial_{v'} | n \rangle. \quad (\text{B7})$$

In general this phase could be brought into our formalism by doing a $U(1)$ gauge transformation $a_n(t) \rightarrow a_n(t) \exp(-i\gamma_n(t))$ and changing $\theta_n \rightarrow \theta_n + \gamma_n(t)$ in (B3) and (B5).

For a slow quench, $v_t \ll 1$. Given that our system is initially prepared in the ground state $n = 0$, so that $a_0(0) = 1$ and $\alpha_n(0) = 0$ for $n \geq 1$. In the leading order of v_t , Eq. (B5) becomes

$$a_n(v) \approx - \int_{v_i}^v dv' \langle n | \partial_{v'} | 0 \rangle e^{i(\theta_n(v') - \theta_0(v'))}. \quad (\text{B8})$$

Now considering for a moment the one-dimensional oscillator $y = e^{i\omega g}$ we use the fact that y satisfies the differential equation

$$y'(x) = i\omega g'(x)y(x) = A(x)y(x). \quad (\text{B9})$$

The asymptotic expansion follows from writing y as $A^{-1}y'$, assuming that $A(x) \neq 0$ in the interval of integration, and integrating by parts

$$\int_a^b y dx = \int_a^b f A^{-1} y' dx = [f A^{-1} y]_a^b - \int_a^b (f A^{-1})' y dx = \frac{1}{i\omega} \left(\frac{f(b)}{g'(b)} y(b) - \frac{f(a)}{g'(a)} y(a) \right) - \frac{1}{i\omega} \int_a^b \left(\frac{f}{g'} \right)' y dx. \quad (\text{B10})$$

Where the notation A^{-1} means matrix (or scalar) inverse, not function inverse. The first term in the right-hand side of Eq. (B10) approximates the integral with an error

$$- \frac{1}{i\omega} \int_a^b \left(\frac{f}{g'} \right)' y dx = \mathcal{O}(\omega^{-2}), \quad (\text{B11})$$

using the fact that the integral decays like $\mathcal{O}(\omega^{-1})$ [33], such that the stand evaluation of a fast oscillating integral, $\int_a^b f(x)e^{i\omega g(x)}dx = \frac{1}{i\omega} \frac{f(x)}{g'(x)} e^{i\omega g(x)} \Big|_a^b + \mathcal{O}(\omega^{-2})$, we obtain

$$\begin{aligned} a_n &\simeq i v_t \frac{\langle n | \partial_v | 0 \rangle}{E_n - E_0} e^{i(\theta_n(v) - \theta_0(v))} \Big|_{v_i}^{v_f} + \mathcal{O}(v_t^2) = -i v_t \frac{\langle n | \partial_v \hat{H} | 0 \rangle}{(E_n - E_0)^2} e^{i(\theta_n(v) - \theta_0(v))} \Big|_{v_i}^{v_f} + \mathcal{O}(v_t^2) \\ &= -i \theta_t \frac{\langle n | \partial_\theta \hat{H} | 0 \rangle}{(E_n - E_0)^2} e^{i(\theta_n(\theta) - \theta_0(\theta))} \Big|_{\theta_i}^{\theta_f} + \mathcal{O}(\theta_t^2) \approx -i \theta_t \frac{\langle n | \partial_\theta \hat{H} | 0 \rangle}{(E_n - E_0)^2} e^{-i\theta_{n0}(\theta)} \Big|_{\theta_i}^{\theta_f}. \end{aligned} \quad (\text{B12})$$

Where $v_t = \theta_t$, and θ belongs to the parameter set v . θ_{n0} is the full phase difference (including the dynamical and the Berry phase) between the n -th and the ground instantaneous eigenstates during time evolution. We note that Eqs. (B6) and (B7) satisfy the $U(1)$ gauge transformation, then we get

$$\begin{aligned} \theta_{n0}(v) &= (\theta_n(v') + \gamma_n(v')) - (\theta_0(v') + \gamma_0(v')) \\ &= \int_{v_i}^{v_f} dv' \left(\frac{E_n(v') - E_0(v')}{v_t'} - (\mathcal{A}_n(v') - \mathcal{A}_0(v')) \right) = \int_{\theta_i}^{\theta_f} d\theta' \left(\frac{E_n(\theta') - E_0(\theta')}{\theta_t'} - (\mathcal{A}_n(\theta') - \mathcal{A}_0(\theta')) \right), \end{aligned} \quad (\text{B13})$$

where \mathcal{A}_n is the Berry-Simon connection equals $i\langle n | \partial_{v'} | n \rangle$.

If the initial state has a large gap or if the protocol is designed in such a way that the initial evolution is adiabatic, then Eq. (B12) takes a particularly simple form

$$a_n \approx -i \theta_t \frac{\langle n | \partial_\theta \hat{H} | 0 \rangle}{(E_n - E_0)^2} \Big|_{\theta_f}. \quad (\text{B14})$$

The contribution of the initial term in Eq. (B12) to the expectation value of the off-diagonal observables can be additionally suppressed by the fast oscillating phase θ_{n0} . From Eq. (B14), using Eq. (4.2), it is straightforward to derive the general force

$$F_\phi = -\langle \psi(t) | \partial_\phi \hat{H} | \psi(t) \rangle$$

$$\begin{aligned} &\approx -\langle 0 | \partial_\phi \hat{H} | 0 \rangle \\ &\quad - \theta_t \text{Im} \sum_{n \neq 0} \frac{\langle 0 | \partial_\theta \hat{H} | n \rangle \langle n | \partial_\phi \hat{H} | 0 \rangle - (\theta \leftrightarrow \phi)}{(E_n - E_0)^2} + \mathcal{O}(\theta_t^2) \\ &= \text{const} + \theta_t F_{\theta\phi} + \mathcal{O}(\theta_t^2), \end{aligned} \quad (\text{B15})$$

where the leading term, the Born-Oppenheimer force, is a constant and the second term is the desired first-order reaction. $\mathcal{O}(\theta_t^2)$ indicates high-order terms, that lead to derivations of the trajectory of the observables of the system.

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